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Where does Flavour Mixing come from?

J.L. Chkareuli

*International Centre for Theoretical Physics, 34100 Trieste, Italy and
Institute of Physics, Georgian Academy of Sciences, 380077 Tbilisi, Georgia*

C.D. Froggatt

*Department of Physics and Astronomy
Glasgow University, Glasgow G12 8QQ, Scotland*

Abstract

We argue that flavour mixing, both in the quark and charged lepton sector, is basically determined by the lightest family mass generation mechanism. So, in the chiral symmetry limit when the up and down quark masses vanish, all the quark mixing angles vanish. This mechanism is not dependent on the number of quark-lepton families nor on any “vertical” symmetry structure, unifying quarks and leptons inside a family as in Grand Unified Theories. Together with a hypothesis of maximal CP violation, the model leads to a completely predictive ansatz for all the CKM matrix elements in terms of the quark masses. Some implications for neutrino masses and oscillations are briefly discussed.

1 Introduction

The pattern of flavour mixing and its relation to the quark-lepton masses is one of the major outstanding problems of particle physics (for a recent review see [1]). Many attempts have been made to interpret this pattern in terms of various family symmetries—discrete or continuous, global or local. Among them, the abelian $U(1)$ [2, 3, 4] and/or non-abelian $SU(2)$ [5, 6, 7] and $SU(3)$ [8, 9] chiral family symmetries seem the most promising. They provide some guidance to the expected hierarchy between the elements of the quark-lepton mass matrices and to the presence of texture zeros [10] in them, leading to relationships between the mass and mixing parameters. In the framework of the supersymmetric Standard Model, such a family symmetry should at the same time provide an almost uniform mass spectrum for the superpartners with a high degree of flavour conservation [11] that makes their existence even more necessary in the SUSY case.

Despite some progress in understanding the flavour mixing problem, one has the uneasy feeling that, in many cases, the problem seems just to be transferred from one place to another. The peculiar quark-lepton mass hierarchy is replaced by a peculiar set of $U(1)$ flavour charges or a peculiar hierarchy of Higgs field VEVs in the non-abelian symmetry case. As a result there are not so many distinctive and testable generic predictions, strictly relating the flavour mixing angles to the quark-lepton masses.

A commonly accepted framework for discussing the flavour problem is based on the picture that, in the absence of flavour mixing, only the particles belonging to the third generation t , b and τ have non-zero masses. All other (physical) masses and the mixing angles appear as a result of the tree-level mixings of families, related to some underlying family symmetry breaking. They might be proportional to powers of some small parameter λ , which are determined by the dimensions of the family symmetry allowed operators that generate the effective (diagonal and off-diagonal) Yukawa couplings for the lighter families in the framework of the (ordinary or supersymmetric) Standard Model.

Using a similar philosophy, we here suggest another possibility:

- Flavour mixing is correlated with the mechanism for generating the masses of the first family and is completely absent in the chiral symmetry limit $m_u = m_d = 0$ (and $m_e = 0$). Therefore, the masses (more

precisely any of the diagonal elements of the quark and charged lepton mass matrices) of the second and third families are practically the same in the gauge (unrotated) and physical bases.

This simple picture can readily be generalised to any number of quark-lepton families, and does not depend on a “vertical” symmetry structure unifying quarks and leptons inside a family as in Grand Unified Theories. In section 2 we present the model motivated by this picture, and the resulting predictions for the quark mixing angles in terms of the quark masses. Assuming a maximal form of CP violation, the CKM matrix is shown to be completely determined and is numerically evaluated in section 3. The implications for lepton mixing and neutrino oscillations are briefly discussed in section 4. Finally we present our conclusions in section 5.

2 The Model

The proposed flavour mixing mechanism driven solely by the generation of the lightest family mass (hereafter called the Lightest Flavour Mixing mechanism or LFM mechanism) could actually be realized in two generic ways.

The first way is when the lightest family mass (m_u , m_d or m_e) appears as a result of the complex flavour mixing of all three families. It “runs along the main diagonal” of the corresponding 3×3 mass matrix M , from the basic dominant element M_{33} to the element M_{22} (via a rotation in the 2-3 sub-block of M) and then to the primordially texture zero element M_{11} (via a rotation in the 1-2 sub-block). The direct flavour mixing of the first and third families of quarks and leptons is supposed to be absent or negligibly small in M .

The second way, on the contrary, presupposes direct flavour mixing of just the first and third families. There is no involvement of the second family in the mixing. In this case, the lightest mass appears in the primordially texture zero M_{11} element “walking round the corner” (via a rotation in the 1-3 sub-block of the mass matrix M). Certainly, this second version of the LFM mechanism cannot be used for both the up and the down quark families simultaneously, since mixing with the second family members is a basic part of the CKM phenomenology (Cabibbo mixing, non-zero V_{cb} element, CP violation). However, this second way could work for the up quark family provided that the down quarks follow the first way.

So, there are two scenarios for the LFM mechanism to be considered.

Scenario A: “ m_u and m_d running along the diagonal”

We propose that the three mass matrices for the Dirac fermions—the up quarks ($U = u, c, t$), the down quarks ($D = d, s, b$) and charged leptons ($E = e, \mu, \tau$)—in the Standard Model, or supersymmetric Standard Model, are each hermitian with three texture zeros of the following form:

$$M_i = \begin{pmatrix} 0 & a_i & 0 \\ a_i^* & A_i & b_i \\ 0 & b_i^* & B_i \end{pmatrix} \quad i = U, D, E \quad (1)$$

It is, of course, necessary to assume some hierarchy between the elements, which we take to be: $B_i \gg A_i \sim |b_i| \gg |a_i|$. We do not attempt to derive this hierarchical structure here. The zeros in the $(M_i)_{11}$ elements correspond to our, and the commonly accepted, conjecture that the lightest family masses appear as a direct result of flavour mixings. The zeros in $(M_i)_{13}$ mean that only minimal “nearest neighbour” interactions occur, giving a tridiagonal matrix structure.

Now our main hypothesis, that the second and third family diagonal mass matrix elements are practically the same in the gauge and physical quark-lepton bases, means that :

$$\vec{B} = (m_t, m_b, m_\tau) + \vec{\delta} \quad \vec{A} = (m_c, m_s, m_\mu) + \vec{\delta}' \quad (2)$$

Here we use the vector notation $\vec{A} = (A_U, A_D, A_E)$ etc. The components δ_i and δ'_i are supposed to be much less than the masses of the particles in the next lightest family, meaning the second and first families respectively:

$$|\vec{\delta}| \ll (m_c, m_s, m_\mu) \quad |\vec{\delta}'| \ll (m_u, m_d, m_e) \quad (3)$$

Since the trace and determinant of the hermitian matrix M_i gives the sum and product of its eigenvalues, it follows that

$$\vec{\delta} \simeq -(m_u, m_d, m_e) \quad (4)$$

while the δ'_i are vanishingly small and can be neglected in further considerations.

It may easily be shown that our hypothesis and related equations (2 - 4) are entirely equivalent to the condition that the diagonal elements (A_i , B_i), of the mass matrices M_i , are proportional to the modulus square of the off-diagonal elements (a_i , b_i):

$$\frac{A_i}{B_i} = \left| \frac{a_i}{b_i} \right|^2 \quad i = U, D, E \quad (5)$$

In this paper we leave aside the question of deriving this proportionality condition, eq. (5), from some underlying theory beyond the Standard Model (see discussion in section 5) and proceed to calculate expressions for all the elements of the matrices M_i and the corresponding CKM quark mixing matrix, in terms of the physical masses.

Using the conservation of the trace, determinant and sum of principal minors of the hermitian matrices M_i under unitary transformations, we are led to a complete determination of the moduli of all their elements. The results can be expressed to high accuracy as follows:

$$\vec{A} = (m_c, m_s, m_\mu) \quad \vec{B} = (m_t - m_u, m_b - m_d, m_\tau - m_e) \quad (6)$$

$$|\vec{a}| = (\sqrt{m_u m_c}, \sqrt{m_d m_s}, \sqrt{m_e m_\mu}) \quad (7)$$

$$|\vec{b}| = (\sqrt{m_u m_t}, \sqrt{m_d m_b}, \sqrt{m_e m_\tau}) \quad (8)$$

As to the CKM matrix V , we must first choose a parameterisation appropriate to our picture of flavour mixing. Among many possible ones, the original Euler parameterisation recently advocated [1, 12] is most convenient:

$$V = \begin{pmatrix} c_U & s_U & 0 \\ -s_U & c_U & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_D & -s_D & 0 \\ s_D & c_U & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} s_U s_D c + c_U c_D e^{-i\phi} & s_U c_D c - c_U s_D e^{-i\phi} & s_U s \\ c_U s_D c - s_U c_D e^{-i\phi} & c_U c_D c + s_U s_D e^{-i\phi} & c_U s \\ -s_D s & -c_D s & c \end{pmatrix} \quad (10)$$

Here $s_{U, D} \equiv \sin \theta_{U, D}$ and $c_{U, D} \equiv \cos \theta_{U, D}$ parameterise simple rotations $R_{12}^{U, D}$ between the first and second families for the up and down quarks respectively, while $s \equiv \sin \theta$ and $c \equiv \cos \theta$ parameterise a rotation between the second and third families. This representation of V takes into account

the observed hierarchical structure of the quark masses and mixing angles. The CP violating phase is connected directly to the first and second families alone.

The quark mass matrices M_U and M_D are diagonalised by unitary transformations which can be written in the form:

$$V_U = R_{12}^U R_{23}^D \Phi^U \quad V_D = R_{12}^D R_{23}^D \Phi^D \quad (11)$$

where $\Phi^{U, D}$ are phase matrices, depending on the phases of the off-diagonal elements $a_i = |a_i| e^{i\alpha_i}$ and $b_i = |b_i| e^{i\beta_i}$:

$$\Phi^U = \begin{pmatrix} e^{i\alpha_U} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\beta_U} \end{pmatrix} \quad \Phi^D = \begin{pmatrix} e^{i\alpha_D} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\beta_D} \end{pmatrix} \quad (12)$$

The CKM matrix is defined by

$$V = V_U V_D^\dagger = R_{12}^U R_{23}^U \Phi^U (\Phi^D)^* (R_{23}^D)^{-1} (R_{12}^D)^{-1} \quad (13)$$

and, after a suitable re-phasing of the quark fields, we can use the representation

$$R_{23}^U \Phi^U (\Phi^D)^* (R_{23}^D)^{-1} = \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \quad (14)$$

The rotation matrices $R_{23}^{U, D}$ and $R_{12}^{U, D}$ for our mass matrices are readily calculated and the CKM matrix expressed in terms of quark mass ratios

$$s_U = \sqrt{\frac{m_u}{m_c}} \quad s_D = \sqrt{\frac{m_d}{m_s}} \quad (15)$$

$$s = \left| \sqrt{\frac{m_d}{m_b}} - e^{i\gamma} \sqrt{\frac{m_u}{m_t}} \right| \quad (16)$$

and two phases $\phi = \alpha_D - \alpha_U$ and $\gamma = \beta_D - \beta_U$.

It follows that the Cabibbo mixing is given by the well-known Fritzsche formula [13]

$$|V_{us}| \simeq |s_U - s_D e^{-i\phi}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \quad (17)$$

which fits the experimental value well, provided that the CP violating phase ϕ is required to be close to $\frac{\pi}{2}$. So, in the following, we shall assume maximal CP violation in the form $\phi = \frac{\pi}{2}$, as is suggested by spontaneous CP violation in the framework of $SU(3)$ family symmetry [8, 14]. The other phase γ appearing in V_{cb} and V_{ub}

$$|V_{cb}| \simeq s \quad |V_{ub}| = s_U s \quad (18)$$

can be rather arbitrary, since the contribution $\sqrt{\frac{m_u}{m_t}}$ to s is relatively small, even compared with the uncertainties coming from the light quark masses themselves. This leads to our most interesting prediction (with the mass ratios calculated at the electroweak scale [15]):

$$|V_{cb}| \simeq \sqrt{\frac{m_d}{m_b}} = 0.038 \pm 0.007 \quad (19)$$

in good agreement with the current data $|V_{cb}| = 0.039 \pm 0.003$ [16]. For definiteness we shall assume the phase γ in s , see eq. (16), to be aligned with the CP violating phase ϕ , again as suggested by $SU(3)$ family symmetry [8, 14], and take $\gamma = \frac{\pi}{2}$. This has the effect of reducing the uncertainty in our prediction eq. (19) from 0.007 to 0.004. Another prediction for the ratio:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\frac{m_u}{m_c}} \quad (20)$$

is quite general for models with “nearest-neighbour” mixing [1].

Scenario B: “ m_u walking around the corner, while m_d runs along the diagonal”

Now the mass matrices for the down quarks M_D and charged leptons M_E are supposed to have the same form as in eq. (1), while the hermitian mass matrix for the up quarks is taken to be:

$$M_U = \begin{pmatrix} 0 & 0 & c_U \\ 0 & A_U & 0 \\ c_U^* & 0 & B_U \end{pmatrix} \quad (21)$$

All the elements of M_U can again be readily determined in terms of the physical masses as:

$$A_U = m_c \quad B_U = m_t - m_u \quad |c_U| = \sqrt{m_u m_t} \quad (22)$$

The quark mass matrices are diagonalised again by unitary transformations as in eq. (11), provided that the matrix V_U is changed to

$$V_U = R_{13}^U \Phi^U \quad (23)$$

where the 1-3 plane rotation of the u and t quarks and the phase matrix Φ^U (depending on the phase of the element $c_U = |c_U| e^{i\alpha_U}$) are parameterised in the following way:

$$R_{13}^U = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \quad \Phi^U = \begin{pmatrix} e^{i\alpha_U} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (24)$$

Here $s_{13} \equiv \sin \theta_{13}$ and $c_{13} \equiv \cos \theta_{13}$.

The structure of the CKM matrix now differs from that of eq. (13) as it contains the direct 1-3 plane rotation for the up quarks:

$$V = V_U V_D^\dagger = R_{13}^U \Phi^U (\Phi^D)^* (R_{23}^D)^{-1} (R_{12}^D)^{-1} \quad (25)$$

although the phases and rotations associated with the down quarks are left the same as before. This natural parameterisation is now quite close to the standard one [16]. The proper mixing angles and CP violating phase (after a suitable re-phasing of the c quark, $c \rightarrow ce^{-i\beta_D}$) are given by the simple and compact formulae:

$$|V_{us}| \simeq s_{12} = \sqrt{\frac{m_d}{m_s}} \quad |V_{cb}| \simeq s_{23} = \sqrt{\frac{m_d}{m_b}} \quad |V_{ub}| \simeq s_{13} = \sqrt{\frac{m_u}{m_t}} \quad (26)$$

and

$$\delta = \alpha_D + \beta_D - \alpha_U \quad (27)$$

While the values of $|V_{us}|$ and $|V_{cb}|$ are practically the same as in scenario A and in good agreement with experiment, a new prediction for $|V_{ub}|$ (not depending on the value of the CP violating phase) should allow experiment to differentiate between the two scenarios in the near future.

3 The CKM Matrix

Our numerical results for both versions of our model, with a maximal CP violating phase (see discussion in section 5), are summarized in the following CKM matrix:

$$V_{CKM} = \begin{pmatrix} 0.975(1) & 0.222(4) & 0.0023(5) & A \\ & & 0.0036(6) & B \\ 0.222(4) & 0.975(1) & 0.038(4) & \\ 0.009(2) & 0.038(4) & 0.999(1) & \end{pmatrix} \quad (28)$$

The uncertainties in brackets are largely given by the uncertainties in the quark masses. There is clearly a real and testable difference between scenarios A and B given by the value of the V_{ub} element. Agreement with the experimental values of the already known CKM matrix elements [16] looks quite impressive. The distinctive predictions for the presently relatively poorly known V_{ub} and V_{td} elements should be tested in the near future, when the B-factories start giving results [16].

4 The Lepton Sector

The lepton mixing matrix is defined analogously to the CKM matrix:

$$U = U_\nu U_E^\dagger \quad (29)$$

Our model predicts the contributions of the charged lepton mixings U_E to the neutrino mixing angles with high accuracy:

$$\sin \theta_{e\mu} = \sqrt{\frac{m_e}{m_\mu}} \quad \sin \theta_{\mu\tau} = \sqrt{\frac{m_e}{m_\tau}} \quad \sin \theta_{e\tau} \simeq 0 \quad (30)$$

These small contributions to the mixing angles from the charged lepton sector will not markedly effect atmospheric neutrino oscillations [17], which appear to require essentially maximal mixing $\sin^2 2\theta_{\mu\tau} \simeq 1$. However $\sin \theta_{e\mu}$ could be relevant to the small angle MSW solution to the solar neutrino problem. Nonetheless our picture of flavour mixing disfavors some recently suggested models (see, e.g. [18]) using a considerable charged lepton mixing contribution to accommodate the atmospheric and solar neutrino data.

It follows that the large neutrino mixing responsible for atmospheric neutrino oscillations should come from the U_ν matrix associated with the neutrino mass matrix. This requires a different mass matrix texture for the neutrinos compared to the charged fermions [19]. So, if some universal family symmetry breaking pattern [14] underlying the LFM mechanism leads to a U_ν matrix with small mixing angles similar to the quarks, it will be necessary to introduce another mechanism to generate neutrino masses. An attractive method for generating a large neutrino mixing within the supersymmetric Standard Model is via R-parity violating interactions, which can give radiatively induced neutrino masses and mixing angles in just the area required by the current observational data [20, 21].

5 Conclusions

The present observational status of quark flavour mixing, as described by the CKM matrix elements [16], shows that the third family t and b quarks are largely decoupled from the lighter families. At first sight, it looks quite surprising that not only the 1-3 “far neighbour” mixing (giving the V_{ub} element in V_{CKM}) but also the 2-3 “nearest neighbour” mixing (V_{cb}) happen to be small compared with the “ordinary” 1-2 Cabibbo mixing (V_{us}) which is determined, according to common belief, by the lightest u and d quarks. This led us to the idea that all the other mixings, and primarily the 2-3 mixing, could also be controlled by the masses m_u and m_d and that the above-mentioned decoupling of the third family t and b quarks is determined by the square roots of the corresponding mass ratios $\sqrt{\frac{m_u}{m_t}}$ and $\sqrt{\frac{m_d}{m_b}}$ respectively. So, in the chiral symmetry limit $m_u = m_d = 0$, not only CP violation vanishes, as argued in [1], but all the flavour mixings disappear as well.

In such a way the Lightest Flavour Mixing (LFM) mechanism, extended also to the charged lepton sector, was formulated in section 2 with two possible scenarios A and B. We found that the LFM mechanism reproduces well the values of the already well measured CKM matrix elements and gives distinctive predictions for the yet poorly known ones in both cases A and B. One could say that, for the first time, there are compact working formulae (especially compact in scenario B) for all the CKM angles in terms of quark mass ratios. The only unknown parameter is the CP violating phase δ . Taking it to be maximal ($\delta = \frac{\pi}{2}$, see below) we obtain a full determination of the

CKM matrix that is numerically summarized in section 3. The LFM mechanism extended to the lepton sector leads, as discussed in section 4, to very small mixings from the charged leptons. If the tree level contributions to the mixings from the neutrino sector are also small, as suggested by the $SU(3)$ family symmetry breaking picture linked to LFM in [14], it is necessary to introduce another neutrino mass generation mechanism to accommodate the recent Super-Kamiokande data [17]. Within the supersymmetric Standard Model, this could well be through radiatively induced neutrino masses and mixings.

From the theoretical point of view the LFM mechanism is based on the generic proportionality condition eq. (5) between diagonal and off-diagonal elements of the mass matrices. For the N family case, this condition could be expressed as:

$$M_{22} : M_{33} : \dots : M_{NN} = |M_{12}|^2 : |M_{23}|^2 : \dots : |M_{N-1\ N}|^2 \quad (31)$$

showing clearly that the heavier families (4th, 5th, ...), had they existed, would be more and more decoupled from the lighter ones and from each other.

One might think that the condition eq. (31) suggests some underlying flavour symmetry, probably non-abelian $SU(N)$, treating the N families in a special way. Indeed, for $N = 3$ families, we have found [14] that the $SU(3)$ chiral family (or horizontal) symmetry [8], properly interpreted in terms of the symmetry breaking vacuum configurations, can lead to the basic condition eq. (5) for the mass matrices of the up and down quarks and charged leptons, depending on the horizontal scalar fields taken.

At the same time the symmetry-breaking horizontal scalar fields, triplets and sextets of $SU(3)$, develop in general complex VEVs and in cases linked to the LFM mechanism transmit a maximal CP violating phase $\delta = \frac{\pi}{2}$ to the effective Yukawa couplings [14]. Apart from the direct predictability of δ (which was used in the numerical analysis of the CKM matrix given in section 3), the possibility that CP symmetry is broken spontaneously like other fundamental symmetries of the Standard Model seems very attractive—both aesthetically and because it gives some clue to the flavour part of strong CP violation. On the other hand, spontaneous CP violation means that the scale of the $SU(3)$ family symmetry must be rather high (not much less than M_{GUT}) in order to avoid the standard domain wall problem by the well-known inflation mechanism [11].

So, an $SU(3)$ family symmetry seems to be a good candidate for the basic theory underlying our proposed LFM mechanism, although we do not exclude the possibility of other interpretations as well. Certainly, even without a theoretical derivation of eq. (5), the LFM mechanism can be considered as a successful predictive ansatz in its own right. Its further testing could shed light on the underlying flavour dynamics and the way towards the final theory of flavour.

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